

THE UNIVERSITY OF AUCKLAND

SEMESTER ONE, 2017

Campus: City

PHILOSOPHY

Modal Logic

(Time allowed: TWO hours)

NOTE: ANSWER EVERY QUESTION FOR A TOTAL OF **50 MARKS**.

Write your answers in the spaces provided. The backs of the exam paper can be used for additional working. An appendices booklet has been provided for reference.

Surname Forenames

University of Auckland ID Number

EXAMINER'S	Question	1	2	3	4	5	6	7	8	9	Total
USE ONLY	Mark										

1. **(7 marks)** Using a tableau, determine whether the following formula is valid in intuitionistic logic, and specify a counter-model if it is invalid.

[illegible]

The formula is valid in intuitionistic logic. _____

ID:.....

2. (3 marks) Which of the following interpretations provides a counter-model to show that

$$\not\models_I (p \supset q) \supset (\neg p \vee q)$$

<p>a)</p> <p>\curvearrowright w_0 $p+$ $q-$</p>	<p>b)</p> <p>\curvearrowright w_0 $p-$ $q-$</p> <p>\nearrow</p> <p>\curvearrowright w_1 $p-$ $q+$</p> <p>\searrow</p> <p>\curvearrowright w_2 $p+$ $q+$</p>
<p>c)</p> <p>\curvearrowright w_0 $p-$ $q-$</p> <p>\nearrow</p> <p>\curvearrowright w_1 $p-$ $q+$</p> <p>\searrow</p> <p>\curvearrowright w_2 $p+$ $q-$</p>	<p>d)</p> <p>\curvearrowright w_0 $p-$ $q-$</p> <p>\rightarrow</p> <p>\curvearrowright w_1 $p+$ $q+$</p>

ID:.....

3. (4 marks) Take a many-valued logic with $V = \{1, 2, 3, 4\}$ and $D = \{2, 3, 4\}$, and two connectives α and β such that:

- For every $x \in V$, $f_\alpha(x) = 1$ if $x = 4$, and $f_\alpha(x) = x + 1$ if $x \leq 3$.
- For every $x, y \in V$, $f_\beta(x, y) = y$ if $x = 1$, and $f_\beta(x, y) = x - 1$ if $x \geq 2$.

- (a) Write down the truth-tables for f_α and f_β .

- (b) Based on your truth-tables, what is the truth-table for the formula

$$\alpha(p \beta p)$$

- (c) True or false?

The formula is valid. _____

4. **(3 marks)** Write down the truth-table for the following formula. Use the truth-tables for the many-valued logic K_3 or LP (remember that the truth tables are the same for K_3 and LP).

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

The formula is valid in LP . _____

5. **(6 marks)** Using a tableau, determine whether the following inference is valid in FDE , and specify a counter-model if it is invalid.

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

The formula is valid in FDE . _____

6. (6 marks)

$$p \supset q, q \supset r \vdash p \supset r$$
This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

The inference is valid in FDE . _____

The inference is valid in K_3 . _____

Page 7 of 10

7. **(7 marks)** Using a tableau, determine whether the following inference is valid in N_* , and specify a counter-model if it is invalid.

[illegible]

The inference is valid in N_* . _____

8. (7 marks) Using a tableau, determine whether the following formula is valid in I_4 , and specify a counter-model if it is invalid.

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

The formula is valid in I_4 . _____

The inference is valid in I_3 . _____

9. (7 marks) Using a tableau, determine whether the following inference is valid in B , and specify a counter-model if it is invalid.

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

The inference is valid in B . _____