QUESTION/ANSWER BOOKLET PHIL 216

THE UNIVERSITY OF AUCKLAND

SEMESTER ONE, 2017
Campus: City

PHILOSOPHY

Modal Logic

(Time allowed: TWO hours)

NOTE: ANSWER EVERY QUESTION FOR A TOTAL OF **50 MARKS.**

Write your answers in the spaces provided. The backs of the exam paper can be used for additional working. An appendices booklet has been provided for reference.

Surname ...Girard............ **Forenames** ...Patrick...........

University of Auckland ID Number ....

EXAMINER'S	Question	1	2	3	4	5	6	7	8	9	Total
USE ONLY	Mark										

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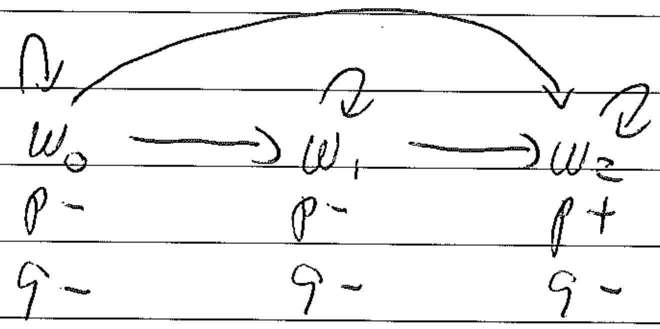
1. **(7 marks)** Using a tableau, determine whether the following formula is valid in intuitionistic logic, and specify a counter-model if it is invalid.

$$\vdash \neg(p \wedge q) \sqsupset (\neg p \wedge \neg q)$$

1.	$\neg(p\wedge q) \sqsupset (\neg p \wedge \neg q)$ ✓	-0	NC
2.	$0r1$ $1r1$		ρ
3.	$\neg(p\wedge q)$ /2	$+1$	$1\sqsupset-$
4.	$\neg p \wedge \neg q$ ✓	-1	$1\sqsupset-$
5.	$\neg p$ -1 ✓ $\neg q$ -1		$4\wedge-$
6.	$1r2$ $2r2$		ρ
7.	p $+2$		$5\neg-$
8.	$p\wedge q$ -2 ✓		$3,6,7+$
9.	p -2 × q -2 ↑		$8\wedge-$



True or false?

The formula is valid in intuitionistic logic. false

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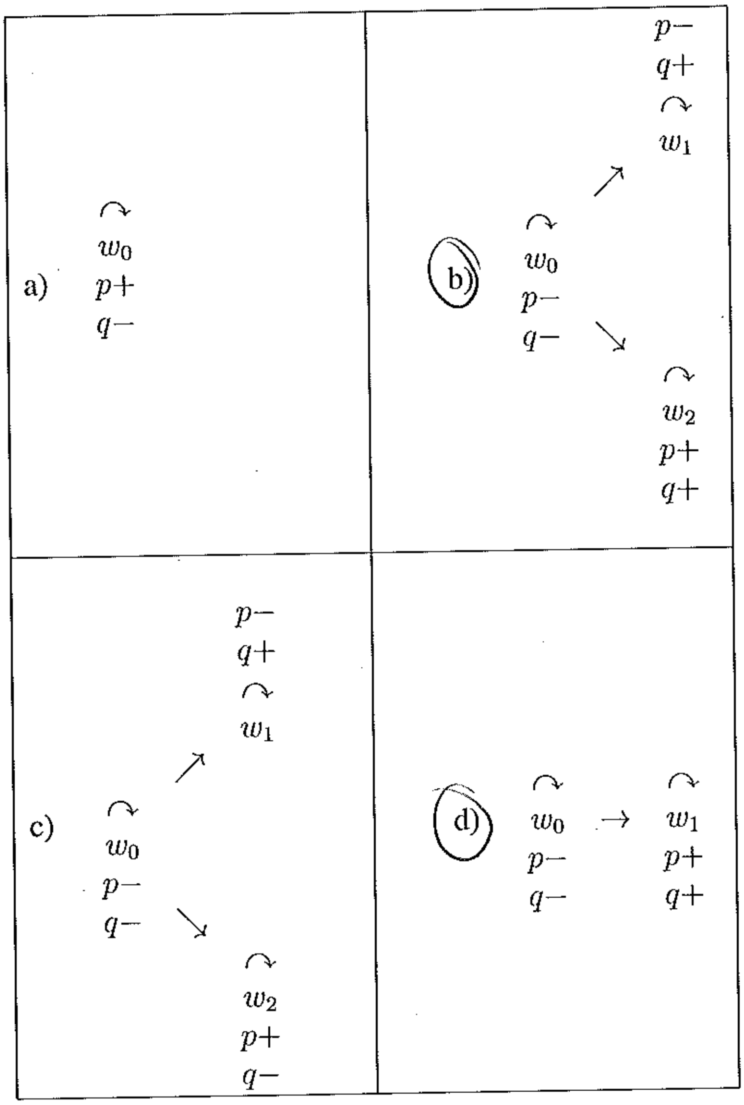
2. **(3 marks)** Which of the following interpretations provides a counter-model to show that

$$\not\models_I (p \supset q) \supset (\neg\, p \vee q)$$



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3. **(4 marks)** Take a many-valued logic with $V = \{1, 2, 3, 4\}$ and $D = \{2, 3, 4\}$, and two connectives α and β such that:

- For every $x \in V$, $f_\alpha(x) = 1$ if $x = 4$, and $f_\alpha(x) = x + 1$ if $x \leq 3$.
- For every $x, y \in V$, $f_\beta(x, y) = y$ if $x = 1$, and $f_\beta(x, y) = x - 1$ if $x \geq 2$.

(a) Write down the truth-tables for f_α and f_β.

	α
1	2
2	3
3	4
4	1

β	1	2	3	4
1	1	2	3	4
2	1	1	1	1
3	2	2	2	2
4	3	3	3	3

(b) Based on your truth-tables, what is the truth-table for the formula

$$\alpha(p\,\beta\,p)$$

p	α	(p	β	p)
1	2	1	1	1
2	2	2	1	2
3	3	3	2	3
4	4	4	3	4

(c) **True or false?**

The formula is valid. yes

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4. **(3 marks)** Write down the truth-table for the following formula. Use the truth-tables for the many-valued logic K_3 or LP (remember that the truth tables are the same for K_3 and LP).

$$(p \wedge \neg p) \supset (q \vee \neg q)$$

p	q	(p	∧	¬	p)	⊃	(q	∨	¬	q)
1	1	1	0	0	1	1	1	1	0	1
1	i	1	0	0	1	i	i	i	i	i
1	0	1	0	0	1	1	0	1	1	0
i	1	i	i	i	i	1	1	1	0	1
i	i	i	i	i	i	i	i	i	i	i
i	0	i	i	i	i	1	0	i	1	0
0	1	0	0	1	0	1	1	1	0	1
0	i	0	0	1	0	1	i	i	i	i
0	0	0	0	1	0	1	0	1	1	0

True or false?

The formula is valid in K_3. false

The formula is valid in LP. true

5. **(6 marks)** Using a tableau, determine whether the following inference is valid in FDE, and specify a counter-model if it is invalid.

$$p \supset r, q \supset r \vdash (p \vee q) \supset r$$

1.	$\neg p \vee r$ ✓	+	P1
2.	$\neg q \vee r$ ✓	+	P2
3.	$\neg(p \vee q) \vee r$ ✓	−	NC
4.	$\neg(p \vee q)$ ✓	−	3 ∨ −
5.	r	−	3 ∨ −
6.	$\neg p \wedge \neg q$ ✓	−	4 ¬∨ −
7.	$\neg p+$ $r+$ ×		1 ∨ +
8.	$\neg q+$ $r+$ ×		2 ∨ +
9.	$\neg p-$ $\neg q-$ × ×		6 ∧ −

True or false?

The formula is valid in FDE. True

6. **(6 marks)**

Write a complete tableau for the following inference, using the tableau rules for FDE. If the inference is invalid, provide a counter-model.

$$p \supset q, q \supset r \vdash p \supset r$$

1.	$\neg p \vee q$ ✓	$+$	P1
2.	$\neg q \vee r$ ✓	$+$	P2
3.	$\neg p \vee r$ ✓	$-$	Nc
4.	$\neg p$	$-$	$3 \vee -$
5.	r	$-$	$3 \vee -$
6.	$\neg p +$ (×)	$q +$	$1 \vee +$
7.	$\neg q +$ ↑	$r +$ (×)	$2 \vee +$

counter-example: qp1 qp0

True or false?

The inference is valid in FDE. false

Remember that the tableau rules for K_3 and LP are the same as for FDE, except for the closure rules. Based on the above tableau, answer the following questions:

True or false?

The inference is valid in K_3. true

The inference is valid in LP. false

7. **(7 marks)** Using a tableau, determine whether the following inference is valid in N_*, and specify a counter-model if it is invalid.

$$(p \wedge q) \rightarrow r \vdash p \rightarrow (\neg q \vee r)$$

1.	$(p \wedge q) \rightarrow r$ / 1, 1#	$+0$	P1
2.	$p \rightarrow (\neg q \vee r)$ ✓	-0	NC
3.	p	$+1$	$2 \rightarrow -$
4.	$\neg q \vee r$ ✓	-1	$2 \rightarrow -$
5.	$\neg q$ ✓	-1	$4 \vee -$
6.	r	-1	$4 \vee -$
7.	q	$+1\#$	$5 \neg -$
8.	$p \wedge q$ -1 ✓ r $+1$ X		$1 \rightarrow +$
9.	p -1 X q -1		$8 \wedge -$
10.	$p \wedge q$ $-1\#$ ✓ r $+1\#$		$1 \rightarrow +$
11.	p $-1\#$ ↑ q $-1\#$ X		$10 \wedge -$

w_0

$\boxed{w_1}$: $p = 1$, $q = 0$, $r = 0$

$\boxed{w_1^*}$: $q = 1$, $r = 1$

True or false?

The inference is valid in N_*. false

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8. **(7 marks)** Using a tableau, determine whether the following formula is valid in I_4, and specify a counter-model if it is invalid.

$$\vdash \neg(p \supset q) \supset (p \supset \neg q)$$

True or false?

The formula is valid in I_4. ____________________

Remember that the tableau rules for I_3 are the same as for I_4, except for the closure rules. Based on the above tableau, answer the following questions:

True or false?

The inference is valid in I_3. ____________________

9. **(7 marks)** Using a tableau, determine whether the following inference is valid in B, and specify a counter-model if it is invalid.

$$(p \wedge q) \rightarrow r \vdash (p \wedge \neg r) \rightarrow \neg q$$

True or false?

The inference is valid in B. ____________________